

Deformed supergravity with local R -symmetry

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Abstract

Using deformation theory based on BRST cohomology, a supergravity model is constructed which interpolates through a continuous deformation parameter between new minimal supergravity with an extra $U(1)$ gauge multiplet and standard supergravity with local R -symmetry in a formulation with a nonstandard set of auxiliary fields. The deformation implements an electromagnetic duality relating the extra $U(1)$ to the R -symmetry. A consistent representative of the R -anomaly in the model is proposed too.

1 Introduction

This paper reports a result that arose from a BRST-cohomological analysis [1] of so-called “old minimal” and “new minimal” supergravity (SUGRA) [2, 3], including their coupling to Yang–Mills gauge multiplets. One of the questions addressed in [1] was the classification of the possible nontrivial consistent deformations of these models. Such deformations may change simultaneously the Lagrangian and the form and algebra of the gauge transformations in a continuous manner, such that the deformed action is invariant under the deformed gauge transformations. A deformation is called trivial if it represents merely a local field redefinition.

Of course, both old and new minimal SUGRA, like most gauge theories, have infinitely many nontrivial deformations. However, most of these deformations change only the action but not the gauge transformations [1]. Such deformations thus add further terms to the action which are invariant under the original gauge transformations, such as invariants involving higher powers in the curvatures and their derivatives.

They are of interest, for instance, within a conventional perturbative quantization approach, for they provide invariant candidate counterterms. The classification and construction of all these terms is described in [1].

On the other hand, deformations which do change nontrivially the gauge transformations are rather exceptional. For instance, they do not exist at all in old minimal SUGRA coupled (only) to Yang–Mills gauge multiplets whenever the corresponding gauge group is semisimple [1]. Of course it is worthwhile to look for deformations which change the gauge transformations in a nontrivial way, for they might provide novel classical SUGRA models, or even occur as quantum deformations of known classical models.

One of these exceptional deformations is presented in this paper. It was announced already in [1] and has rather unusual features. Namely, it deforms new minimal SUGRA into standard (“old”) minimal SUGRA with local R -symmetry. More precisely, it involves a deformation parameter, denoted by g_1 throughout the paper, such that $g_1 = 0$ reproduces new minimal SUGRA with an extra local $U(1)$ symmetry (different from the R -symmetry), whereas for all nonvanishing values of g_1 the deformed model is equivalent to old minimal SUGRA with local R -symmetry, at least classically. The latter equivalence holds on-shell, after suitable local field redefinitions which make sense only for $g_1 \neq 0$ and provide a new set of auxiliary fields closing the supersymmetry (SUSY) algebra off-shell.

The resulting model thus has the remarkable property to incorporate two different SUGRA models and to connect them *continuously* by means of a coupling constant (deformation parameter). The deformation exists thanks to the presence of a 2-form gauge potential in new minimal SUGRA and requires the coupling of new minimal SUGRA to an extra $U(1)$ gauge multiplet. The extra $U(1)$ symmetry disappears effectively for $g_1 \neq 0$ via an electromagnetic duality which relates it to the R -symmetry and is implemented by the deformation itself. It might be instructive to examine whether this result can be related to the standard constructions [4, 5] relating old and new minimal SUGRA by seeming different duality transformations.

The paper has been organized as follows. Section 2 first reviews briefly the construction [6] of consistent deformations by BRST cohomological means in general, and then illustrates the specific computation performed here for a very simple toy model discussed already in [1]. The description of the technically more involved cal-

ulation in SUGRA is relegated to the appendix. Section 3 presents the resulting deformation of new minimal SUGRA coupled only to the extra $U(1)$ gauge multiplet. The relation of this model to standard (old minimal) SUGRA with local R -symmetry is established in section 4 which also contains a discussion of the above-mentioned electromagnetic duality. In section 5 extensions of the model are discussed, namely the inclusion of a Fayet–Iliopoulos term [7] for the extra $U(1)$ symmetry and the coupling to matter and further Yang–Mills gauge multiplets. Finally, in section 6, the question of anomalies associated with the R -symmetry of our model is addressed and a consistent (= BRST invariant) representative of “the” R -anomaly is proposed in explicit form. I use the same conventions as in [1].

2 Sketch of the computation

2.1 Consistent deformations and BRST cohomology

The relation of consistent deformations of gauge theories to the BRST cohomology was pointed out by Barnich and Henneaux in [6]. It is based on the field-antifield formalism of Batalin and Vilkovisky [8]. The idea is to deform the solution of the (classical) master equation which is the central quantity of this formalism. To that end one departs from the solution $\mathcal{S}^{(0)}$ of the master equation in the original (undeformed) theory and looks for a deformed solution \mathcal{S} of the form

$$\mathcal{S} = \mathcal{S}^{(0)} + g\mathcal{S}^{(1)} + \frac{1}{2}g^2\mathcal{S}^{(2)} + \dots \quad (2.1)$$

where g is a deformation parameter which plays the role of a coupling constant in the deformed theory. The master equation for \mathcal{S} ,

$$(\mathcal{S}, \mathcal{S}) = 0, \quad (2.2)$$

is then decomposed into parts with definite degrees in g , leading to a tower of equations which are investigated one after another,

$$(\mathcal{S}^{(0)}, \mathcal{S}^{(0)}) = 0, \quad (\mathcal{S}^{(0)}, \mathcal{S}^{(1)}) = 0, \quad (\mathcal{S}^{(1)}, \mathcal{S}^{(1)}) + (\mathcal{S}^{(0)}, \mathcal{S}^{(2)}) = 0, \quad \dots \quad (2.3)$$

The first of these equations is the master equation for the original (undeformed) model. The second one, together with the nontriviality of the sought deformations, requires $\mathcal{S}^{(1)}$ to represent a nontrivial cohomology class of the local BRST cohomology

in the original theory. More precisely, $\mathcal{S}^{(1)}$ is determined by $H^0(s^{(0)})$, the cohomology of the original (undeformed) BRST operator $s^{(0)}$ on local functionals of the fields and antifields at ghost number zero¹. The subsequent equations in (2.3) can further obstruct the construction of \mathcal{S} through $H^1(s^{(0)})$, the cohomology of $s^{(0)}$ at ghost number one. For instance, the third equation (2.3) requires $(\mathcal{S}^{(1)}, \mathcal{S}^{(1)})$ to be trivial (BRST-exact) in $H^1(s^{(0)})$ (it is $s^{(0)}$ -closed by the second equation (2.3), thanks to the Jacobi identity for the antibracket).

A study of the BRST cohomology at ghost numbers zero and one therefore allows to classify and construct systematically the various consistent deformations of a given gauge theory. (2.1) is of course only a special case of deformations of the form

$$\mathcal{S} = \mathcal{S}^{(0)} + \sum_i g_i \mathcal{S}_i^{(1)} + \frac{1}{2} \sum_{ij} g_i g_j \mathcal{S}_{ij}^{(2)} + \dots \quad (2.4)$$

which can be analysed analogously to (2.3). In particular, the $\mathcal{S}_i^{(1)}$ represent inequivalent cohomology classes of $H^0(s^{(0)})$.

In our case, $\mathcal{S}^{(0)}$ is of the form

$$\mathcal{S}^{(0)} = \int d^4x \left(\mathcal{L}^{(0)} - (s^{(0)} \Phi^A) \Phi_A^* \right) \quad (2.5)$$

where $\mathcal{L}^{(0)}$ and $s^{(0)}$ are the Lagrangian and the BRST operator for new minimal SUGRA and Φ_A^* denotes the antifields. The deformed solution of the master equation will still depend only linearly on the antifields, i.e. the gauge algebra will be closed off-shell even in the deformed theory and \mathcal{S} will take the form

$$\mathcal{S} = \int d^4x \left(\mathcal{L} - (s \Phi^A) \Phi_A^* \right) \quad (2.6)$$

where \mathcal{L} and $s \Phi^A$ will not involve antifields.

2.2 Toy model revisited

Explicit computations in SUGRA are often rather involved. Appropriate methods that simplify the calculations are therefore very welcome. In this section I illustrate the technique used to derive the results of this paper for a toy model discussed already in [1]. The analogous calculation in the SUGRA case is sketched in appendix A.1.

¹ $s^{(0)}$ is strictly nilpotent on all fields and antifields and generated in the antibracket by $\mathcal{S}^{(0)}$ according to $s^{(0)} \cdot = (\mathcal{S}^{(0)}, \cdot)$. Analogously the BRST operator in the deformed theory is denoted by s and generated by \mathcal{S} according to $s \cdot = (\mathcal{S}, \cdot)$.

The toy model is defined in flat four dimensional Minkowski space by the Lagrangian

$$\mathcal{L}^{(0)} = -2\epsilon^{\mu\nu\rho\sigma} a_\mu \partial_\nu t_{\rho\sigma} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad (2.7)$$

where a_μ and $t_{\mu\nu}$ are the components of an abelian gauge field and 2-form gauge potential respectively, and F is the field strength of a second abelian gauge field,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . \quad (2.8)$$

The toy model is evidently invariant under gauge transformations corresponding to the following simple BRST transformations:

$$\begin{aligned} s^{(0)} t_{\mu\nu} &= \partial_\nu Q_\mu - \partial_\mu Q_\nu , & s^{(0)} Q_\mu &= \partial_\mu Q , & s^{(0)} Q &= 0 , \\ s^{(0)} a_\mu &= \partial_\mu c , & s^{(0)} c &= 0 , \\ s^{(0)} A_\mu &= \partial_\mu C , & s^{(0)} C &= 0 \end{aligned} \quad (2.9)$$

where Q_μ , c and C are the ghost fields corresponding to $t_{\mu\nu}$, a_μ and A_μ respectively, and Q is a ghost for the ghosts Q_μ (i.e. Q has ghost number two). Due to the closure of the gauge algebra, the proper solution $\mathcal{S}^{(0)}$ of the master equation corresponding to (2.7) and (2.9) is just of the form (2.5). According to the standard rules of the field-antifield formalism, the BRST transformations of the antifields are then obtained from

$$s^{(0)} \Phi_A^* = \left(\mathcal{S}^{(0)}, \Phi_A^* \right) = \frac{\delta^R \mathcal{S}^{(0)}}{\delta \Phi^A} . \quad (2.10)$$

Consider now the following total forms (= formal sums of local differential forms):

$$\tilde{C}^* = d^4 x C^* + \frac{1}{6} dx^\mu dx^\nu dx^\rho \epsilon_{\mu\nu\rho\sigma} A^{\sigma*} + \frac{1}{4} dx^\mu dx^\nu \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} , \quad (2.11)$$

$$\tilde{Q} = Q + dx^\mu Q_\mu + \frac{1}{2} dx^\mu dx^\nu t_{\mu\nu} , \quad (2.12)$$

$$H = \frac{1}{2} dx^\mu dx^\nu dx^\rho \partial_\mu t_{\nu\rho} \quad (2.13)$$

where C^* and $A^{\mu*}$ are the antifields of C and A_μ , the differentials dx^μ are treated as Grassmann odd (anticommuting) quantities, and

$$d^4 x = dx^0 dx^1 dx^2 dx^3 = -\frac{1}{24} \epsilon_{\mu\nu\rho\sigma} dx^\mu dx^\nu dx^\rho dx^\sigma .$$

It is easy to verify that \tilde{C}^* and \tilde{Q} satisfy

$$\tilde{s}^{(0)} \tilde{C}^* = 0 , \quad \tilde{s}^{(0)} \tilde{Q} = H \quad (2.14)$$

where $\tilde{s}^{(0)}$ is the sum of $s^{(0)}$ and the spacetime exterior derivative $d = dx^\mu \partial_\mu$,

$$\tilde{s}^{(0)} = s^{(0)} + d . \quad (2.15)$$

(2.14) implies evidently

$$\tilde{s}^{(0)}(\tilde{C}^* \tilde{Q}) = \tilde{C}^* H = 0 \quad (2.16)$$

where the second equality holds because $\tilde{C}^* H$ contains only form degrees exceeding four. (2.16) decomposes of course into the so-called descent equations $s^{(0)}\omega_4 + d\omega_3 = 0$, $s^{(0)}\omega_3 + d\omega_2 = 0$, $s^{(0)}\omega_2 = 0$ satisfied by the p -forms ω_p contained in $\tilde{C}^* \tilde{Q} = \sum \omega_p$ ($p = 2, 3, 4$). In particular it thus implies that $\int \omega_4$ is $s^{(0)}$ -invariant (up to the boundary term $\int d(-\omega_3)$). Furthermore, $\int \omega_4$ is cohomological nontrivial, because its antifield independent part does not vanish on-shell up to a boundary term, and is thus a candidate first order deformation which reads explicitly

$$\mathcal{S}^{(1)} = \int d^4x \left(-\frac{1}{2} F^{\mu\nu} t_{\mu\nu} + A^{\mu*} Q_\mu + C^* Q \right) . \quad (2.17)$$

It is straightforward to verify that, dropping a boundary term,

$$(\mathcal{S}^{(1)}, \mathcal{S}^{(1)}) = 2 \int d^4x Q_\nu \partial_\mu t^{\mu\nu} = s^{(0)} \int d^4x \frac{1}{2} t_{\mu\nu} t^{\mu\nu} . \quad (2.18)$$

Hence, the first three equations (2.3) are satisfied with $\mathcal{S}^{(1)}$ as in (2.17) and

$$\mathcal{S}^{(2)} = - \int d^4x \frac{1}{2} t_{\mu\nu} t^{\mu\nu} . \quad (2.19)$$

Evidently $\mathcal{S}^{(1)}$ and $\mathcal{S}^{(2)}$ satisfy

$$(\mathcal{S}^{(1)}, \mathcal{S}^{(2)}) = (\mathcal{S}^{(2)}, \mathcal{S}^{(2)}) = 0 . \quad (2.20)$$

We thus conclude that a deformed solution of the master equation is given by

$$\begin{aligned} \mathcal{S} &= \mathcal{S}^{(0)} + g \mathcal{S}^{(1)} + \frac{1}{2} g^2 \mathcal{S}^{(2)} \\ &= \int d^4x \left\{ -2\epsilon^{\mu\nu\rho\sigma} a_\mu \partial_\nu t_{\rho\sigma} - \frac{1}{4} (F^{\mu\nu} + g t^{\mu\nu})(F_{\mu\nu} + g t_{\mu\nu}) \right. \\ &\quad \left. + t^{\mu\nu*} (\partial_\nu Q_\mu - \partial_\mu Q_\nu) - Q^{\mu*} \partial_\mu Q + a^{\mu*} \partial_\mu C \right. \\ &\quad \left. + A^{\mu*} (\partial_\mu C + g Q_\mu) + g C^* Q \right\} . \end{aligned} \quad (2.21)$$

From this one reads off easily the deformed Lagrangian and BRST transformations of the fields.

3 Simplest model

This section presents the result obtained by deforming new minimal SUGRA coupled only to one $U(1)$ -gauge multiplet analogously to the toy model. This SUGRA model has $(16+16)$ degrees of freedom off-shell². The field content, including the ghost fields, is given in the table below which also indicates the ghost numbers (gh), Grassmann parities (ε), dimension assignments (dim), R -charges (r) and reality properties of the fields.

| Φ | $gh(\Phi)$ | $\varepsilon(\Phi)$ | $dim(\Phi)$ | $r(\Phi)$ | |
|--------------|------------|---------------------|-------------|-----------|--------------------------------|
| e_μ^a | 0 | 0 | 0 | 0 | vierbein fields (real) |
| C^μ | 1 | 1 | -1 | 0 | diffeomorphism ghosts (real) |
| C^{ab} | 1 | 1 | 0 | 0 | Lorentz ghosts (real) |
| ψ_μ | 0 | 1 | 1/2 | 1 | gravitino (complex) |
| ξ | 1 | 0 | -1/2 | 1 | SUSY ghosts (complex) |
| $t_{\mu\nu}$ | 0 | 0 | 0 | 0 | 2-form gauge potential (real) |
| Q_μ | 1 | 1 | -1 | 0 | ghosts for $t_{\mu\nu}$ (real) |
| Q | 2 | 0 | -2 | 0 | ghost for ghosts (imaginary) |
| a_μ | 0 | 0 | 1 | 0 | R -gauge field (real) |
| c | 1 | 1 | 0 | 0 | R -ghost (real) |
| A_μ | 0 | 0 | 1 | 0 | $U(1)$ gauge field (real) |
| λ | 0 | 1 | 3/2 | 1 | $U(1)$ gaugino (complex) |
| D | 0 | 0 | 2 | 0 | scalar aux. field (real) |
| C | 1 | 1 | 0 | 0 | $U(1)$ ghost (real) |

The model has the following gauge symmetries: general coordinate and local Lorentz invariance, local N=1 SUSY, local R -symmetry, the local $U(1)$ symmetry associated with A_μ , and the reducible gauge symmetry associated with $t_{\mu\nu}$. The corresponding BRST transformations are given explicitly below. The action contains two separately invariant parts. Their integrands are denoted by eL_{grav} and $eL_{U(1)}$ and occur with coefficients M_{Pl}^2 and g_0^{-2} respectively, where M_{Pl} is the Planck mass and g_0 is a coupling constant for the $U(1)$ symmetry associated with A_μ (g_0 may be absorbed by

²We employ here the usual counting where one counts separately the fermionic and bosonic fields (including the auxiliary ones), subtracting respectively the number of gauge symmetries, and adding the number of reducibility conditions.

rescaling A_μ , λ , D , C and g_1). The deformation parameter g_1 has the same dimension as M_{Pl}^2 , i.e. $g_1 M_{Pl}^{-2}$ is dimensionless. The Lagrangian reads

$$\mathcal{L} = e \left(M_{Pl}^2 L_{grav} + g_0^{-2} L_{U(1)} \right), \quad (3.1)$$

$$\begin{aligned} L_{grav} = & \frac{1}{2} R - 2\varepsilon^{\mu\nu\rho\sigma} (\psi_\mu \sigma_\nu \nabla_\rho \bar{\psi}_\sigma - \bar{\psi}_\mu \bar{\sigma}_\nu \nabla_\rho \psi_\sigma) \\ & - 3H_\mu H^\mu - 2\varepsilon^{\mu\nu\rho\sigma} a_\mu \partial_\nu t_{\rho\sigma}, \end{aligned} \quad (3.2)$$

$$\begin{aligned} L_{U(1)} = & -\frac{1}{4} (F_{\mu\nu} + g_1 t_{\mu\nu}) (F^{\mu\nu} + g_1 t^{\mu\nu}) + \frac{1}{2} D^2 - \frac{1}{8} g_1^2 \\ & - \frac{1}{2} i (\lambda \sigma^\mu \nabla_\mu \bar{\lambda} + \bar{\lambda} \bar{\sigma}^\mu \nabla_\mu \lambda) + \frac{1}{2} g_1 (i \lambda \sigma^\mu \bar{\psi}_\mu - i \psi_\mu \sigma^\mu \bar{\lambda}) \\ & - \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} (F_{\mu\nu} + g_1 t_{\mu\nu}) (\psi_\rho \sigma_\sigma \bar{\lambda} + \lambda \sigma_\sigma \bar{\psi}_\rho) + \frac{3}{2} \lambda \sigma^\mu \bar{\lambda} H_\mu \\ & + \psi_\mu \sigma^{\mu\nu} \psi_\nu \bar{\lambda} \bar{\lambda} + \bar{\psi}_\mu \bar{\sigma}^{\mu\nu} \bar{\psi}_\nu \lambda \lambda \end{aligned} \quad (3.3)$$

with

$$e = \det(e_\mu^a), \quad (3.4)$$

$$\varepsilon^{\mu\nu\rho\sigma} = E_a^\mu \dots E_d^\sigma \varepsilon^{abcd} = e^{-1} \epsilon^{\mu\nu\rho\sigma} \quad (\epsilon^{0123} = 1), \quad (3.5)$$

$$R = 2E_a^\nu E_b^\mu (\partial_{[\mu} \omega_{\nu]}^{ab} - \omega_{[\mu}^{ca} \omega_{\nu]c}^b), \quad (3.6)$$

$$H^\mu = \varepsilon^{\mu\nu\rho\sigma} (\frac{1}{2} \partial_\nu t_{\rho\sigma} + i \psi_\nu \sigma_\rho \bar{\psi}_\sigma), \quad (3.7)$$

$$F_{\mu\nu} = 2(\partial_{[\mu} A_{\nu]} + i \lambda \sigma_{[\mu} \bar{\psi}_{\nu]} + i \psi_{[\mu} \sigma_{\nu]} \bar{\lambda}), \quad (3.8)$$

$$\nabla_\mu \psi_\nu = \partial_\mu \psi_\nu - \frac{1}{2} \omega_\mu^{ab} \psi_\nu \sigma_{ab} - i a_\mu \psi_\nu, \quad (3.9)$$

$$\nabla_\mu \lambda = \partial_\mu \lambda - \frac{1}{2} \omega_\mu^{ab} \lambda \sigma_{ab} - i a_\mu \lambda. \quad (3.10)$$

where E_a^μ and ω_μ^{ab} denote the components of the inverse vielbein and of the standard gravitino dependent spin connection respectively,

$$E_a^\mu e_\nu^a = \delta_\nu^\mu, \quad E_a^\mu e_\mu^b = \delta_a^b, \quad (3.11)$$

$$\begin{aligned} \omega_\mu^{ab} &= E^{a\nu} E^{b\rho} (\omega_{[\mu\nu]\rho} - \omega_{[\nu\rho]\mu} + \omega_{[\rho\mu]\nu}), \\ \omega_{[\mu\nu]\rho} &= e_{\rho a} \partial_{[\mu} e_{\nu]}^a - i \psi_\mu \sigma_\rho \bar{\psi}_\nu + i \psi_\nu \sigma_\rho \bar{\psi}_\mu. \end{aligned} \quad (3.12)$$

The deformed BRST transformations of the classical fields are:

$$\begin{aligned} s e_\mu^a &= C^\nu \partial_\nu e_\mu^a + (\partial_\mu C^\nu) e_\nu^a + C_b^a e_\mu^b \\ &+ 2i(\xi \sigma^a \bar{\psi}_\mu - \psi_\mu \sigma^a \bar{\xi}), \end{aligned} \quad (3.13)$$

$$\begin{aligned} s \psi_\mu &= C^\nu \partial_\nu \psi_\mu + (\partial_\mu C^\nu) \psi_\nu + \frac{1}{2} C^{ab} \psi_\mu \sigma_{ab} + i c \psi_\mu \\ &+ \partial_\mu \xi - \frac{1}{2} \omega_\mu^{ab} \xi \sigma_{ab} - i a_\mu \xi - i \xi H_\mu - i \xi \sigma_{\mu\nu} H^\nu, \end{aligned} \quad (3.14)$$

$$\begin{aligned}
st_{\mu\nu} &= C^\rho \partial_\rho t_{\mu\nu} + (\partial_\mu C^\rho) t_{\rho\nu} + (\partial_\nu C^\rho) t_{\mu\rho} + \partial_\nu Q_\mu - \partial_\mu Q_\nu \\
&\quad - i (\xi \sigma_\mu \bar{\psi}_\nu - \xi \sigma_\nu \bar{\psi}_\mu + \psi_\mu \sigma_\nu \bar{\xi} - \psi_\nu \sigma_\mu \bar{\xi}) ,
\end{aligned} \tag{3.15}$$

$$\begin{aligned}
sa_\mu &= C^\nu \partial_\nu a_\mu + (\partial_\mu C^\nu) a_\nu + \partial_\mu c \\
&\quad + \xi \sigma_\mu \bar{S} + S \sigma_\mu \bar{\xi} ,
\end{aligned} \tag{3.16}$$

$$\begin{aligned}
sA_\mu &= \partial_\mu C + C^\nu \partial_\nu A_\mu + (\partial_\mu C^\nu) A_\nu \\
&\quad - i \xi \sigma_\mu \bar{\lambda} + i \lambda \sigma_\mu \bar{\xi} + g_1 Q_\mu ,
\end{aligned} \tag{3.17}$$

$$\begin{aligned}
s\lambda &= C^\mu \partial_\mu \lambda + \frac{1}{2} C^{ab} \lambda \sigma_{ab} + i c \lambda \\
&\quad + \xi (\frac{1}{2} g_1 - iD) - \xi \sigma^{\mu\nu} (F_{\mu\nu} + g_1 t_{\mu\nu}) ,
\end{aligned} \tag{3.18}$$

$$\begin{aligned}
sD &= C^\mu \partial_\mu D \\
&\quad + \xi \sigma^\mu \left[\nabla_\mu \bar{\lambda} - \bar{\psi}_\mu (iD + \frac{1}{2} g_1) - \bar{\sigma}^{\nu\rho} \bar{\psi}_\mu (F_{\nu\rho} + g_1 t_{\nu\rho}) \right] \\
&\quad + \left[(\nabla_\mu \lambda) + (iD - \frac{1}{2} g_1) \psi_\mu + (F_{\nu\rho} + g_1 t_{\nu\rho}) \psi_\mu \sigma^{\nu\rho} \right] \sigma^\mu \bar{\xi} \\
&\quad + \frac{3}{2} i (\xi \sigma^\mu \bar{\lambda} - \lambda \sigma^\mu \bar{\xi}) H_\mu
\end{aligned} \tag{3.19}$$

where S^α and $\bar{S}^{\dot{\alpha}}$ which occur in (3.16) are the spin- $\frac{1}{2}$ parts of the super-covariant gravitino field strengths,

$$S = 2(\nabla_\mu \psi_\nu) \sigma^{\mu\nu} + \frac{3}{2} i \psi_\mu H^\mu , \quad \bar{S} = -2\bar{\sigma}^{\mu\nu} \nabla_\mu \bar{\psi}_\nu - \frac{3}{2} i \bar{\psi}_\mu H^\mu . \tag{3.20}$$

The BRST transformation of the ghosts and the ghost for ghosts are

$$sC^\mu = C^\nu \partial_\nu C^\mu + 2i \xi \sigma^\mu \bar{\xi} , \tag{3.21}$$

$$s\xi = C^\mu \partial_\mu \xi + \frac{1}{2} C^{ab} \xi \sigma_{ab} + i c \xi - 2i \xi \sigma^\mu \bar{\xi} \psi_\mu , \tag{3.22}$$

$$sC^{ab} = C^\mu \partial_\mu C^{ab} + C^{ca} C_c{}^b - 2i \xi \sigma^\mu \bar{\xi} \omega_\mu{}^{ab} + 2i \varepsilon^{abcd} \xi \sigma_c \bar{\xi} H_d , \tag{3.23}$$

$$sc = C^\mu \partial_\mu c - 2i \xi \sigma^\mu \bar{\xi} a_\mu , \tag{3.24}$$

$$sC = C^\mu \partial_\mu C - 2i \xi \sigma^\mu \bar{\xi} A_\mu - g_1 Q , \tag{3.25}$$

$$sQ_\mu = \partial_\mu Q + C^\nu \partial_\nu Q_\mu + (\partial_\mu C^\nu) Q_\nu - 2i \xi \sigma^\nu \bar{\xi} t_{\mu\nu} - i \xi \sigma_\mu \bar{\xi} , \tag{3.26}$$

$$sQ = C^\mu \partial_\mu Q - 2i \xi \sigma^\mu \bar{\xi} Q_\mu . \tag{3.27}$$

In the formulas (3.13)–(3.27) the spinor indices of ψ_μ , $\bar{\psi}_\mu$, λ , $\bar{\lambda}$, S , \bar{S} , ξ and $\bar{\xi}$ are everywhere upstairs. The above BRST transformations are off-shell nilpotent,

$$s^2 \Phi = 0 \quad \forall \Phi . \tag{3.28}$$

As $s\Phi$ does not involve antifields, the gauge algebra closes off-shell, as promised.

Notice that g_1 appears only in $eL_{U(1)}$ and in the BRST transformations of A_μ , λ , D and C . Notice also that the deformation introduces a cosmological constant and spontaneous SUSY breaking which are not present for $g_1 = 0$. λ is the Goldstone fermion for the broken SUSY. The cosmological constant can be removed when matter multiplets are included, cf. section 5.2. Notice that (3.18) looks as if the D -field has obtained a constant *imaginary* part through the deformation.

For later purpose I note that the equations of motion for ψ_μ , a_μ and $t_{\mu\nu}$ imply the following on-shell equalities (indicated by \approx):

$$S^\alpha \approx -g_1(2g_0M_{Pl})^{-2}\lambda^\alpha, \quad (3.29)$$

$$H^\mu \approx (2g_0M_{Pl})^{-2}\lambda\sigma^\mu\bar{\lambda}, \quad (3.30)$$

$$2\partial_{[\mu}a_{\nu]} \approx g_1(2g_0M_{Pl})^{-2}\{2\lambda\sigma_{[\mu}\bar{\psi}_{\nu]} - 2\psi_{[\mu}\sigma_{\nu]}\bar{\lambda} + \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}(F^{\rho\sigma} + g_1t^{\rho\sigma})\}. \quad (3.31)$$

4 Relation to old minimal SUGRA and duality

It will now be shown that the deformed model of section 3 is classically (on-shell) equivalent to old minimal SUGRA with local R -symmetry for all nonvanishing values of g_1 . In contrast, $g_1 = 0$ gives of course new minimal SUGRA coupled to a $U(1)$ gauge multiplet.

The reason is that for $g_1 \neq 0$ both the deformed action and gauge resp. BRST transformations depend on the $t_{\mu\nu}$ and A_μ only via the combinations $g_1t_{\mu\nu} + \partial_\mu A_\nu - \partial_\nu A_\mu$. For $g_1 \neq 0$, we can therefore introduce these combinations as new elementary fields instead of the $t_{\mu\nu}$ (in fact we will use a slightly different choice below). These new fields become auxiliary and can be eliminated algebraically after redefining also the R -gauge field appropriately. After elimination of the auxiliary fields, it becomes evident that the deformed model is (for $g_1 \neq 0$) indeed classically equivalent to old minimal SUGRA with local R -symmetry. Note however that the auxiliary field content differs from that of old minimal SUGRA: instead of a real vector field and a complex scalar field, the deformed model contains after the field redefinitions an auxiliary real antisymmetric tensor field. It should be kept in mind that the field redefinitions make sense only for $g_1 \neq 0$, i.e. the redefined fields cannot be used to describe the complete model, in contrast to the original fields used in section 3.

Convenient field redefinitions are

$$b_{\mu\nu} = g_0^{-1}g_R(g_1t_{\mu\nu} + F_{\mu\nu}), \quad (4.1)$$

$$\hat{a}_\mu = a_\mu + \frac{3}{4} H_\mu - \frac{3}{16} (g_0 M_{Pl})^{-2} \lambda \sigma_\mu \bar{\lambda} , \quad (4.2)$$

$$\hat{\lambda} = i g_0^{-1} g_R \lambda , \quad (4.3)$$

$$\hat{D} = g_0^{-1} g_R D \quad (4.4)$$

with $F_{\mu\nu}$ and H_μ as in (3.8) and (3.7) respectively, and

$$g_R = \frac{g_1}{4g_0 M_{Pl}^2} . \quad (4.5)$$

g_R is the dimensionless R -coupling constant in the deformed theory. This becomes clear when one writes the deformed Lagrangian (3.1) in terms of the redefined fields:

$$\mathcal{L}_{(g_1 \neq 0)} = e (M_{Pl}^2 L_1 + g_R^{-2} L_2 + g_R^{-2} L_3) \quad (4.6)$$

$$\begin{aligned} L_1 = & \frac{1}{2} R - 2 \varepsilon^{\mu\nu\rho\sigma} (\psi_\mu \sigma_\nu \hat{\nabla}_\rho \bar{\psi}_\sigma - \bar{\psi}_\mu \bar{\sigma}_\nu \hat{\nabla}_\rho \psi_\sigma) \\ & + 2(\hat{\lambda} \sigma^\mu \bar{\psi}_\mu + \psi_\mu \sigma^\mu \bar{\hat{\lambda}}) - 2(g_R M_{Pl})^2 , \end{aligned} \quad (4.7)$$

$$\begin{aligned} L_2 = & -\frac{i}{2} \hat{\lambda} \sigma^\mu \hat{\nabla}_\mu \bar{\hat{\lambda}} + \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \hat{F}_{\mu\nu} \hat{\lambda} \sigma_\rho \bar{\psi}_\sigma \\ & + \bar{\psi}_\mu \bar{\sigma}^{\mu\nu} \bar{\psi}_\nu \hat{\lambda} \hat{\lambda} + \frac{3}{16} (g_R M_{Pl})^{-2} \hat{\lambda} \hat{\lambda} \bar{\hat{\lambda}} \bar{\hat{\lambda}} + c.c. , \end{aligned} \quad (4.8)$$

$$L_3 = \frac{1}{2} \hat{D}^2 - \frac{1}{4} b_{\mu\nu} (b^{\mu\nu} + \varepsilon^{\mu\nu\rho\sigma} \hat{F}_{\rho\sigma}) \quad (4.9)$$

with

$$\hat{F}_{\mu\nu} = 2(\partial_{[\mu} \hat{a}_{\nu]} + i \hat{\lambda} \sigma_{[\mu} \bar{\psi}_{\nu]} + i \psi_{[\mu} \sigma_{\nu]} \bar{\hat{\lambda}}) , \quad (4.10)$$

$$\hat{\nabla}_\mu \psi_\nu = \partial_\mu \psi_\nu - \frac{1}{2} \omega_\mu^{ab} \psi_\nu \sigma_{ab} - i \hat{a}_\mu \psi_\nu , \quad (4.11)$$

$$\hat{\nabla}_\mu \hat{\lambda} = \partial_\mu \hat{\lambda} - \frac{1}{2} \omega_\mu^{ab} \hat{\lambda} \sigma_{ab} - i \hat{a}_\mu \hat{\lambda} . \quad (4.12)$$

As promised, $b_{\mu\nu}$ is an auxiliary field. Its classical equation of motion reads

$$b^{\mu\nu} \approx -\frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \hat{F}_{\rho\sigma} . \quad (4.13)$$

Upon elimination of $b_{\mu\nu}$ and \hat{D} , L_3 becomes $(-\frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu})$ and the complete action turns indeed into the one of old minimal SUGRA with local R -symmetry (see e.g. [1]) after eliminating the auxiliary fields there too.

The BRST transformations for the redefined fields can be obtained from the formulas of section 3. For instance one gets

$$\begin{aligned} s \hat{\lambda} = & C^\mu \partial_\mu \hat{\lambda} + \frac{1}{2} C^{ab} \hat{\lambda} \sigma_{ab} + i c \hat{\lambda} \\ & + \xi (\hat{D} + 2i g_R^2 M_{Pl}^2) - i \xi \sigma^{\mu\nu} b_{\mu\nu} . \end{aligned} \quad (4.14)$$

$\hat{\lambda}$ plays for $g_1 \neq 0$ the role of the gaugino for R -transformations. This is read off from the above formulas, taking into account (4.13) and (3.29). Note that the latter reads in terms of the redefined fields just $S \approx i\hat{\lambda}$ which “explains” why $\hat{\lambda}$ turns into the R -gaugino. Namely S plays in fact in new minimal SUGRA the role of a composite R -gaugino (cf. e.g. appendix B of [1]).

Let me now briefly discuss the electromagnetic duality mentioned in the introduction. It relates, for $g_1 \neq 0$, the $U(1)$ -symmetry associated with A_μ to the R -symmetry and is established by (4.13) (resp. by (3.31)). The latter shows indeed that $b_{\mu\nu}$, which contains the super-covariant $U(1)$ field strength $F_{\mu\nu}$, is (on-shell) dual to the super-covariant R -field strength $\hat{F}_{\mu\nu}$. Moreover, we have just seen that λ , which is for $g_1 = 0$ the gaugino associated with A_μ , turns for $g_1 \neq 0$ on-shell into the R -gaugino. Last but not least, it is striking that the R -coupling constant g_R is proportional to the inverse $U(1)$ -coupling constant g_0 , cf. (4.5).

In this context it is worthwhile to recall that for $g_1 = 0$, i.e. in new minimal SUGRA, there is no R -coupling constant in the usual sense, which reflects that one cannot switch off the R -symmetry in new minimal SUGRA without switching off simultaneously SUSY. Furthermore, on-shell the R -gauge field a_μ is for $g_1 = 0$ pure gauge, at least locally, cf. (3.31). Hence, for $g_1 = 0$ this gauge field does not carry local physical degrees of freedom, in contrast to A_μ . Turning on g_1 , A_μ disappears effectively through the above field redefinitions, and transfers its degrees of freedom via the duality to the (redefined) R -gauge field.

5 Extensions

5.1 Inclusion of a Fayet-Iliopoulos term

We will now discuss extensions of the simple model discussed in section 3. First we include a Fayet-Iliopoulos term for the $U(1)$ symmetry associated with A_μ . This term is introduced by a second deformation of the model and we denote the corresponding deformation parameter by g_2 (it has the same dimension as g_1 and M_{Pl}^2). It turns out that this deformation does not cause further modifications of the gauge resp. BRST transformations and adds only the following term to the Lagrangian which is separately invariant under the BRST transformations given in section 3 up to a total

derivative:

$$\begin{aligned} \mathcal{L}_{FI} = & g_2 e (D + \lambda \sigma^\mu \bar{\psi}_\mu + \psi_\mu \sigma^\mu \bar{\lambda} \\ & + \varepsilon^{\mu\nu\rho\sigma} A_\mu \partial_\nu t_{\rho\sigma} + \tfrac{1}{4} g_1 \varepsilon^{\mu\nu\rho\sigma} t_{\mu\nu} t_{\rho\sigma}) . \end{aligned} \quad (5.1)$$

Evidently this term provides for all values of g_1 an additional contribution to the cosmological constant. Repeating the discussion of section 4, one finds that the inclusion of (5.1) results for $g_1 \neq 0$ again in a model which is classically equivalent to old minimal SUGRA with local R -symmetry. The only differences are shifts of the cosmological constant, the R -coupling constant and the SUSY breaking parameter, and the occurrence of a theta term for the R -symmetry. The latter is of course classically irrelevant. The R -coupling constant is now

$$g'_R = \frac{\sqrt{(g_1/g_0)^2 + (2g_2g_0)^2}}{4M_{pl}^2} . \quad (5.2)$$

It is the modulus of a complex parameter

$$z = \frac{2g_2g_0 + ig_1/g_0}{4M_{pl}^2} = g'_R e^{i\theta}$$

whose phase appears in the coefficient of the above-mentioned theta term for the R -symmetry according to

$$-\tfrac{1}{2} (g'_R)^{-2} \cot \theta \varepsilon^{\mu\nu\rho\sigma} \partial_\mu \hat{a}_\nu \partial_\rho \hat{a}_\sigma$$

with \hat{a}_μ as in (4.2) (note that $\theta = 0$ is excluded here as it corresponds to $g_1 = 0$). (4.2) and field redefinitions analogous to (4.1), (4.3) and (4.4), namely

$$b'_{\mu\nu} = g'_R g_0^{-1} (g_1 t_{\mu\nu} + F_{\mu\nu}) , \quad (5.3)$$

$$\hat{\lambda}' = g'_R g_0^{-1} e^{i\theta} \lambda , \quad (5.4)$$

$$\hat{D}' = g'_R (g_0^{-1} D + g_0 g_2) , \quad (5.5)$$

establish for $g_1 \neq 0$ again the classical equivalence of the model based on the sum of (3.1) and (5.1) to old minimal SUGRA with local R -symmetry. In particular $b'_{\mu\nu}$ becomes again an auxiliary field, and after eliminating it one arrives in fact at an action of exactly the same form as in section 4 after eliminating $b_{\mu\nu}$ there. The only differences are that $\hat{\lambda}$, \hat{D} and g_R are replaced by $\hat{\lambda}'$, \hat{D}' and g'_R respectively, and that the above theta term appears.

Notice that the appearance of the R -coupling constant and the coefficient of the theta term in a single complex parameter z is reminiscent of similar (though somewhat different) relations in other globally and locally supersymmetric gauge theories.

5.2 Coupling to matter and further gauge multiplets

The inclusion of matter or further gauge multiplets is completely straightforward. Indeed, recall that the deformation modifies only the BRST transformations of A_μ , λ , D and C , but not the gauge transformation of any other field. Hence, any term involving matter or further gauge multiplets which is gauge invariant in new minimal SUGRA will be gauge invariant in the deformed theory too, provided it does not involve A_μ , λ or D . Of course this requires in particular that all the matter fields transform trivially under the $U(1)$ symmetry associated with A_μ . The latter requirement however is just a necessary prerequisite for the existence of the deformation in presence of matter fields, as pointed out in [1], and must thus be imposed anyhow.

In particular, gauge invariant contributions to the action containing the kinetic terms for Yang–Mills multiplets, as well as the kinetic and superpotential terms for chiral matter multiplets are thus exactly the same as in new minimal SUGRA. Therefore the cosmological constant implemented by the deformation can be removed by the usual mechanism when matter multiplets are included, cf. [9, 5] and, more recently, [10]. Furthermore, the standard kinetic and superpotential terms for matter and gauge multiplets depend on $t_{\mu\nu}$ only through its super-covariant field strength H_μ given in (3.7). As a shift of $t_{\mu\nu}$ by $\partial_\mu A_\nu - \partial_\nu A_\mu$ drops out in H_μ , one concludes again that the complete action depends for $g_1 \neq 0$ on $t_{\mu\nu}$ and A_μ only through $b_{\mu\nu}$.

6 Anomalies

It is well-known that the presence of a classical R -gauge symmetry can lead to chiral anomalies³, at least in old minimal SUGRA. As the latter is classically equivalent to our model for $g_1 \neq 0$, one expects that the R -symmetry will implement chiral anomalies in our model too, unless all the contributions to these anomalies (by the gravitino, gauginos and matter fermions) cancel. Furthermore, the same argument suggests that these anomaly cancellation conditions coincide with those in old minimal SUGRA analysed recently in [10].

As anomalies correspond to BRST cohomology classes at ghost number one, it is therefore instructive to look for representatives of such cohomology classes which

³Besides the “pure” R -anomaly, these are also “mixed” anomalies, such as mixed gravitational and R -anomalies.

can correspond to chiral anomalies associated with the R -symmetry in our model. In particular “the” (pure) R -anomaly is of interest in this context, as it has some unusual features as compared to chiral anomalies in more standard theories. Namely, inspired by the familiar representatives of chiral anomalies, one might expect that this anomaly is represented by a BRST-invariant functional of the form $\int c da da + \text{“more”}$ where $a = dx^\mu a_\mu$ is the R -connection and “more” indicates terms to be chosen such that the complete expression is invariant under the BRST transformations given in section 3. However, in our case $\int c da da$ vanishes on-shell up to terms of higher order in g_1 , cf. (3.31). Therefore this term can be removed from any BRST-invariant functional by subtracting a cohomologically trivial (BRST-exact) local functional. (3.31) now suggests an alternative expression representing the R -anomaly in our model, namely $\int c B^2 + \text{“more”}$ where B is the 2-form corresponding to (4.1). As shown in appendix A.2, one finds indeed a corresponding BRST-invariant functional which reads in complete form

$$\begin{aligned}
\mathcal{A} = \int d^4x e \bigg\{ & \frac{1}{4} c \varepsilon^{\mu\nu\rho\sigma} (F_{\mu\nu} + g_1 t_{\mu\nu}) (F_{\rho\sigma} + g_1 t_{\rho\sigma}) \\
& + i \varepsilon^{\mu\nu\rho\sigma} (\lambda \sigma_\mu \bar{\xi} - \xi \sigma_\mu \bar{\lambda}) a_\nu (F_{\rho\sigma} + g_1 t_{\rho\sigma}) \\
& + \xi S \bar{\lambda} \bar{\lambda} - 2 \xi \lambda \bar{S} \bar{\lambda} + \bar{S} \bar{\xi} \lambda \lambda - 2 \bar{\lambda} \bar{\xi} S \lambda \\
& - g_1 c (D + \lambda \sigma^\mu \bar{\psi}_\mu + \psi_\mu \sigma^\mu \bar{\lambda}) \\
& + g_1 (\xi \sigma^\mu \bar{\lambda} + \lambda \sigma^\mu \bar{\xi}) a_\mu \bigg\} \tag{6.1}
\end{aligned}$$

with $F_{\mu\nu}$, S and \bar{S} as in (3.8) and (3.20). Note that (6.1) does not depend on antifields and would thus provide a universal representative of the R -anomaly which does not change even in presence of matter or further gauge multiplets. Nevertheless it is instructive to cast it in a different but equivalent form. Namely, adding to it sX with

$$X = \int d^4x e \varepsilon^{\mu\nu\rho\sigma} a_\mu A_\nu (\partial_\rho A_\sigma + g_1 t_{\rho\sigma})$$

and dropping total derivatives in the integrand, one gets (cf. appendix A.2)

$$\begin{aligned}
\mathcal{A} + sX \approx g_1 \int d^4x e \{ & \varepsilon^{\mu\nu\rho\sigma} (c A_\mu - C a_\mu) \partial_\nu t_{\rho\sigma} \\
& - \frac{1}{8} (g_0 M_{Pl})^{-2} C F_{\mu\nu} F^{\mu\nu} + \text{terms with fermions} \} + O(g_1^2) \tag{6.2}
\end{aligned}$$

where we have restricted ourselves to the simple model described in section 3. (6.2) shows that $\mathcal{A} + sX$ contains on-shell at lowest order in g_1 a linear combination of the

two terms $e \varepsilon^{\mu\nu\rho\sigma} (c A_\mu - C a_\mu) \partial_\nu t_{\rho\sigma}$ and $e C F_{\mu\nu} F^{\mu\nu}$ which indeed give rise to nontrivial and inequivalent BRST-invariant functionals in new minimal SUGRA, cf. [1], section 9. This signals (though it does not prove) that \mathcal{A} is cohomologically nontrivial in the deformed theory and can thus indeed represent an anomaly.

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A Details of the calculations

A.1 Deformation

In the following $s^{(0)}$ denotes the BRST operator in new minimal SUGRA obtained from (3.13)–(3.27) for $g_1 = 0$. First one computes the analogue of (2.11) in new minimal SUGRA. This means simply to compute a solution of the descent equations corresponding to $d^4 x C^*$. The existence of such a solution follows from $s^{(0)} C^* = \partial_\mu (C^\mu C^* - A^{\mu*})$. The result can be conveniently expressed in terms of the following quantities which are useful also for the computation itself:

$$\hat{D}^* = e^{-1} D^* , \quad (\text{A.1})$$

$$\hat{\lambda}_\alpha^* = e^{-1} (\lambda^* + \sigma^\mu \bar{\psi}_\mu D^*)_\alpha , \quad (\text{A.2})$$

$$\hat{\bar{\lambda}}_{\dot{\alpha}}^* = e^{-1} (\bar{\lambda}^* - \psi_\mu \sigma^\mu D^*)_{\dot{\alpha}} , \quad (\text{A.3})$$

$$\hat{A}^{a*} = e_\mu^a (e^{-1} A^{\mu*} + 2 \hat{\lambda}^* \sigma^{\mu\nu} \psi_\nu - 2 \bar{\psi}_\nu \bar{\sigma}^{\mu\nu} \hat{\bar{\lambda}}^* + 2 i \varepsilon^{\mu\nu\rho\sigma} \psi_\nu \sigma_\rho \bar{\psi}_\sigma \hat{D}^*) , \quad (\text{A.4})$$

$$\hat{C}^* = e^{-1} C^* , \quad (\text{A.5})$$

$$\tilde{\xi}^a = (C^\mu + dx^\mu) e_\mu^a , \quad (\text{A.6})$$

$$\tilde{\xi}^\alpha = \xi^\alpha + (C^\mu + dx^\mu) \psi_\mu^\alpha , \quad (\text{A.7})$$

$$\tilde{\xi}^{\dot{\alpha}} = \bar{\xi}^{\dot{\alpha}} - (C^\mu + dx^\mu) \bar{\psi}_\mu^{\dot{\alpha}} . \quad (\text{A.8})$$

Without going into details I note that (A.1)–(A.5) are super-covariant (combinations of) antifields in the sense that their BRST transformations do not contain derivatives

of ghosts. They correspond to the super-covariant version of the equations of motion. (A.6)–(A.8) are “generalized connections” used already in [11, 1]. In terms of these quantities, the sought analogue of (2.11) reads

$$\begin{aligned}\tilde{C}^* &= \Xi \hat{C}^* + \frac{1}{6} \tilde{\xi}^a \tilde{\xi}^b \tilde{\xi}^c \varepsilon_{abcd} \hat{A}^{d*} + \frac{1}{2} i \tilde{\xi}^a (\hat{\lambda}^* \sigma_a \bar{\vartheta} + \vartheta \sigma_a \hat{\lambda}^*) \\ &\quad + 2i \Theta \hat{D}^* + \frac{1}{4} \tilde{\xi}^a \tilde{\xi}^b \varepsilon_{abcd} F^{cd} + \vartheta \lambda - \bar{\vartheta} \bar{\lambda}\end{aligned}\quad (\text{A.9})$$

where $F_{ab} = E_a^\mu E_b^\nu F_{\mu\nu}$ with $F_{\mu\nu}$ as in (3.8) and

$$\Xi = -\frac{1}{24} \tilde{\xi}^a \tilde{\xi}^b \tilde{\xi}^c \tilde{\xi}^d \varepsilon_{abcd}, \quad (\text{A.10})$$

$$\vartheta^\alpha = \tilde{\xi}_{\dot{\alpha}} \tilde{\xi}^{\dot{\alpha}\alpha}, \quad \bar{\vartheta}^{\dot{\alpha}} = \tilde{\xi}^{\dot{\alpha}\alpha} \tilde{\xi}_\alpha, \quad \Theta = \tilde{\xi}_\alpha \tilde{\xi}^{\dot{\alpha}\alpha} \tilde{\xi}_{\dot{\alpha}} \quad (\text{A.11})$$

with $\tilde{\xi}^{\dot{\alpha}\alpha} = \tilde{\xi}^a \bar{\sigma}_a^{\dot{\alpha}\alpha}$. The SUGRA-analogues of (2.12) and (2.13) read

$$\tilde{Q} = Q + (C^\mu + dx^\mu) Q_\mu + \frac{1}{2} (C^\nu + dx^\nu) t_{\mu\nu}, \quad (\text{A.12})$$

$$\mathcal{H} = \frac{1}{6} \tilde{\xi}^a \tilde{\xi}^b \tilde{\xi}^c \varepsilon_{abcd} H^d + i \Theta \quad (\text{A.13})$$

with $H^a = H^\mu e_\mu^a$, H^μ as in (3.7). \tilde{C}^* , \tilde{Q} and \mathcal{H} satisfy identities analogous to (2.14),

$$\tilde{s}^{(0)} \tilde{C}^* = 0, \quad \tilde{s}^{(0)} \tilde{Q} = \mathcal{H} \quad (\text{A.14})$$

with $\tilde{s}^{(0)} = s^{(0)} + d$. However, in contrast to (2.16), $\tilde{s}^{(0)}(\tilde{C}^* \tilde{Q}) = \tilde{C}^* \mathcal{H}$ does *not* vanish in the SUGRA case because it contains pieces with degrees 2, 3 and 4 in the $\tilde{\xi}^a$ (the $\tilde{\xi}^a$ play a part analogous to the differentials in the toy model). Therefore we have to seek an $\tilde{s}^{(0)}$ -invariant completion of $\tilde{C}^* \tilde{Q}$. The existence of this completion was proved in [1]. It can be efficiently computed using a technique described in section 4 of [11]. The result is the following $\tilde{s}^{(0)}$ -invariant total form:

$$\omega = \tilde{Q} \tilde{C}^* + \frac{1}{2} i (\eta \lambda - \bar{\eta} \bar{\lambda}) + \frac{1}{2} \Xi (\tilde{\xi}^\alpha \hat{\lambda}_\alpha^* - \tilde{\xi}^{\dot{\alpha}} \hat{\lambda}_{\dot{\alpha}}^*) \quad (\text{A.15})$$

where

$$\eta^\alpha = -\frac{i}{6} \vartheta^\beta \tilde{\xi}_{\beta\dot{\beta}} \tilde{\xi}^{\dot{\beta}\alpha}, \quad \bar{\eta}^{\dot{\alpha}} = \frac{i}{6} \tilde{\xi}^{\dot{\alpha}\beta} \tilde{\xi}_{\beta\dot{\beta}} \bar{\vartheta}^{\dot{\beta}}. \quad (\text{A.16})$$

The volume form ω_4 contained in ω provides $\mathcal{S}^{(1)} = \int \omega_4$. Explicitly one gets

$$\mathcal{S}^{(1)} = \int d^4x e K, \quad (\text{A.17})$$

$$\begin{aligned}K &= -\frac{1}{2} t_{\mu\nu} F^{\mu\nu} + \frac{1}{2} t_{\mu\nu} \varepsilon^{\mu\nu\rho\sigma} (\lambda \sigma_\rho \bar{\psi}_\sigma + \psi_\sigma \sigma_\rho \bar{\lambda}) + \frac{i}{2} (\lambda \sigma^\mu \bar{\psi}_\mu - \psi_\mu \sigma^\mu \bar{\lambda}) \\ &\quad + Q C^* - Q_\mu A^{\mu*} + (\frac{1}{2} \xi - t_{\mu\nu} \xi \sigma^{\mu\nu}) \lambda^* - \bar{\lambda}^* (\frac{1}{2} \bar{\xi} + \bar{\sigma}^{\mu\nu} \bar{\xi} t_{\mu\nu}) \\ &\quad + (\frac{1}{2} \xi \sigma^\mu \bar{\psi}_\mu + \frac{1}{2} \psi_\mu \sigma^\mu \bar{\xi} + t_{\mu\nu} \xi \sigma^\rho \bar{\sigma}^{\mu\nu} \bar{\psi}_\rho - t_{\mu\nu} \psi_\rho \sigma^{\mu\nu} \sigma^\rho \bar{\xi}) D^*. \quad (\text{A.18})\end{aligned}$$

As in the case of the toy model, it is now easy to complete the computation of the deformation. The result is

$$\mathcal{S} = \mathcal{S}^{(0)} + g_1 \mathcal{S}^{(1)} + \frac{1}{2} g_1^2 \mathcal{S}^{(2)} \quad (\text{A.19})$$

with

$$\mathcal{S}^{(2)} = - \int d^4 x e \left(\frac{1}{2} t_{\mu\nu} t^{\mu\nu} + \frac{1}{4} \right). \quad (\text{A.20})$$

Technical remark:

I used the convention that the set $\{\Phi^A\}$ contains λ and $\bar{\lambda}$ with spinor indices *downstairs*. Hence, the sum $(-s\Phi^A)\Phi_A^*$ in (2.6) contains for instance $(-s\lambda_\alpha)\lambda^{\alpha*} = +(s\lambda^\alpha)\lambda_\alpha^*$, and (3.18) and (A.18) are consistent because the spinor indices of $s\lambda$ are *upstairs* in both formulas.

Furthermore I remark that \mathcal{S} is required to be real. In the conventions of [1] used here, this corresponds for instance to the following reality properties of the antifields of λ , $\bar{\lambda}$, D and A_μ :

$$\bar{\lambda}^* = -\overline{\lambda^*}, \quad D^* = -\overline{D^*}, \quad A^{\mu*} = -\overline{A^{\mu*}}.$$

A.2 Candidate anomaly

The results given in section 6 can be derived analogously, using now $\tilde{s} = s + d$ rather than $\tilde{s}^{(0)}$ and in addition the following total forms:

$$\tilde{C} = C + (C^\mu + dx^\mu) A_\mu, \quad (\text{A.21})$$

$$\tilde{c} = c + (C^\mu + dx^\mu) a_\mu, \quad (\text{A.22})$$

$$\begin{aligned} \mathcal{B} = & (C^\mu + dx^\mu)(C^\nu + dx^\nu)(\partial_\mu A_\nu + \frac{1}{2} g_1 t_{\mu\nu}) \\ & -i (C^\mu + dx^\mu)(\lambda \sigma_\mu \bar{\xi} - \xi \sigma_\mu \bar{\lambda}), \end{aligned} \quad (\text{A.23})$$

$$\hat{\mathcal{F}} = (C^\mu + dx^\mu)(C^\nu + dx^\nu) \partial_\mu a_\nu - (C^\mu + dx^\mu)(S \sigma_\mu \bar{\xi} + \xi \sigma_\mu \bar{S}) \quad (\text{A.24})$$

with S and \bar{S} as in (3.20). These forms satisfy

$$\tilde{s}\tilde{c} = \hat{\mathcal{F}}, \quad \tilde{s}\tilde{C} = \mathcal{B} - g_1 \tilde{Q}, \quad \tilde{s}\mathcal{B} = g_1 \tilde{s}\tilde{Q} = \mathcal{H} \quad (\text{A.25})$$

with \tilde{Q} and \mathcal{H} as in (A.12) and (A.13). Using (A.25) and the technique of section 4 of [11], one can check that the following total form is \tilde{s} -invariant:

$$\begin{aligned} \omega_{\mathcal{A}} = & \tilde{c} (\mathcal{B}^2 - g_1 \eta \lambda - g_1 \bar{\eta} \bar{\lambda} - g_1 \Xi D) \\ & + \Xi (\xi S \bar{\lambda} \bar{\lambda} - 2 \xi \lambda \bar{S} \bar{\lambda} + \bar{S} \bar{\xi} \lambda \lambda - 2 \bar{\lambda} \bar{\xi} S \lambda) \end{aligned} \quad (\text{A.26})$$

with Ξ as in (A.10). The volume form contained in $\omega_{\mathcal{A}}$ is just the integrand of (6.1). (6.2) is obtained by means of (3.31) from the volume form contained in

$$\omega_{\mathcal{A}} + \tilde{s}\{\tilde{c}\tilde{C}(\mathcal{B} + g_1\tilde{Q})\} = \hat{\mathcal{F}}\tilde{C}\mathcal{B} + 2g_1\tilde{c}\tilde{C}\mathcal{H} + \dots . \quad (\text{A.27})$$

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